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basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS/ NATIONAL SENIOR CERTIFICATE EXAMINATIONS

MATHEMATICS P1

MAY/JUNE 2024

MARKS: 150

TIME: 3 hours

This question paper consists of 11 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly

QUESTION 1

1.1 Solve for x:

1.1.1 $3x^2 + 5x = 0$ (2)

1.1.2 $4x^2 + 3x - 5 = 0$ (answers correct to TWO decimal places) (3)

1.1.3 $(x-1)^2 - 9 \geq 0$ (4)

1.1.4 $5^{2x} - 5^x = 0$ (4)

1.1.5 $\dfrac{x}{\sqrt{20-x}} = 1$ (5)

1.2 Solve for x and y simultaneously:

$x + y = 9$ and $2x^2 - y^2 = 7$ (5)

1.3 Given: $P = (1-a)$ and $T = (1+a)(1+a^2)(1+a^4)\ldots(1+a^{512})$

Determine the value of $P \times T$ in terms of a. (3)

[26]

QUESTION 2

2.1 Consider the geometric series: $4 + 2 + 1 + \frac{1}{2} + \ldots$

2.1.1 Does this series converge? Justify your answer. (2)

2.1.2 Calculate S_∞. (2)

2.2 Given: $\sum_{p=k}^{10} 3^{p-1} = 29\ 520$. Calculate the value of k. (5)

[9]

QUESTION 3

3.1 Consider the quadratic number pattern: 3 ; 7 ; 12 ; …

3.1.1 Show that the general term of this number pattern is given by $T_n = \frac{1}{2}n^2 + \frac{5}{2}n$. (3)

3.1.2 What number must be added to T_{n-1} so that $T_n = 13\ 527$? (4)

3.2 Given an arithmetic sequence with $T_1 = 8$ and $T_2 = 11$.

3.2.1 Calculate the value of n if $T_n = 41$. (3)

3.2.2 A new arithmetic sequence P is formed using the term position and the term value of the given arithmetic sequence.
For the new sequence, $P_8 = 1$, $P_{11} = 2$ and so forth.

(a) Write down the value of P_{41}. (1)

(b) Calculate the value of the first term of the new arithmetic sequence. (4)
[15]

QUESTION 4

Given: $g(x) = \frac{1}{x-1} + 2$

4.1 Write down the equations of the asymptotes of g. (2)

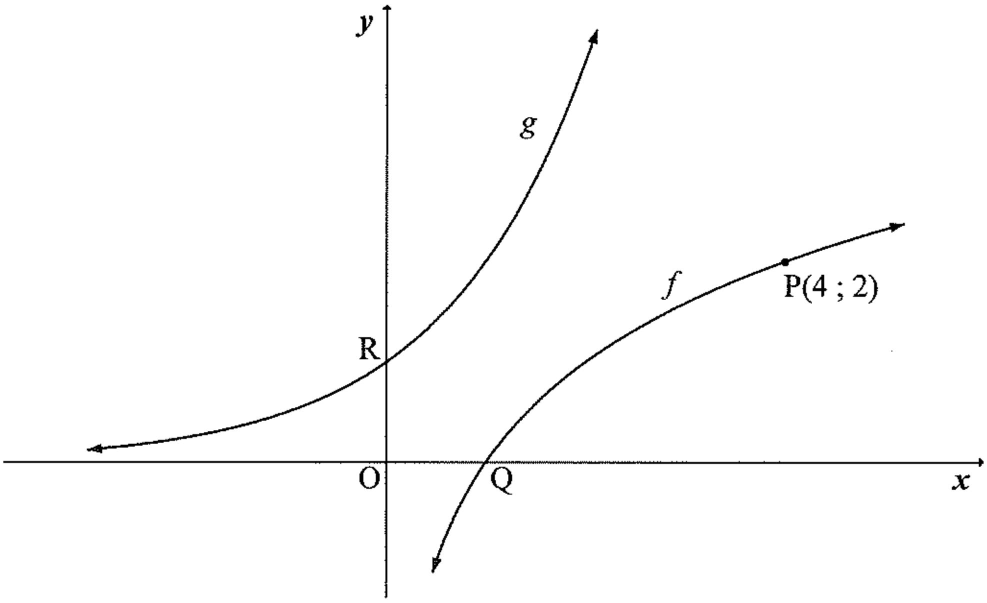
4.2 Draw a graph of g, indicating any intercepts with the axes and asymptotes. (4)

4.3 Determine the values of x where $g(x) > 0$. (2)

4.4 Determine the equation of the axis of symmetry of g which has a negative gradient. (2)
[10]

QUESTION 5

In the diagram, the graphs of $f(x) = \log_a x$ and g are drawn. Graph g is the reflection of f in the line $y = x$. Graph f passes through the point P(4 ; 2). Q is the x-intercept of f and R is the y-intercept of g.



5.1 Write down the coordinates of P$'$, the image of P on g. (2)

5.2 Show that $a = 2$. (2)

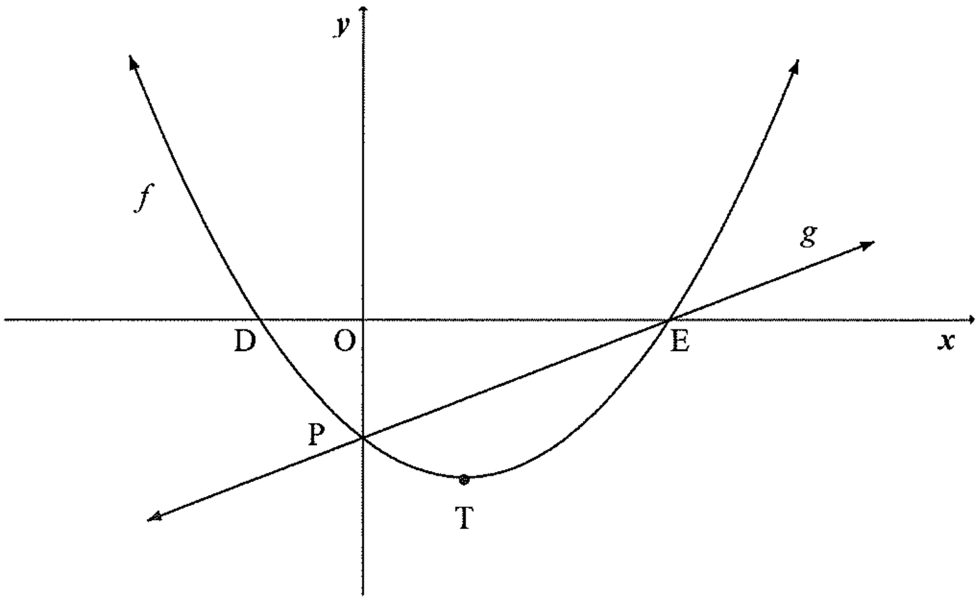
5.3 Write down the equation of g in the form $y = \ldots$ (1)

5.4 T is a point on f in the first quadrant where TR is parallel to the x-axis. Calculate the area of ΔRTP$'$. (4)

[9]

QUESTION 6

The graphs of $f(x) = x^2 - 2x - 3$ and $g(x) = mx + c$ are drawn below. D and E are the x-intercepts and P is the y-intercept of f. The turning point of f is T(1 ; – 4). The graphs of f and g intersect at P and E.



6.1	Write down the range of f.	(1)
6.2	Calculate the coordinates of D and E.	(3)
6.3	Determine the equation of g.	(2)
6.4	Write down the values of x for which $f(x) - g(x) > 0$.	(2)
6.5	Determine the maximum vertical distance between h and g if $h(x) = -f(x)$ for $x \in [-2 ; 3]$.	(5)
6.6	Given: $k(x) = g(x) - n$. Determine n if k is a tangent to f.	(5)

[18]

QUESTION 7

7.1 Six years ago, Thabo bought a phone for R13 000. The value of the phone depreciated annually according to the reducing-balance method. The value of the phone is now R8 337,75. Calculate the annual rate of depreciation. (3)

7.2 Eric and Thandi need to save R80 000 each to go on a holiday at the end of December 2027.

- Thandi decides that she will start saving at the end of January 2025. She will make 36 monthly deposits into a savings account that pays interest at 8,6% p.a., compounded monthly. The deposit will be made at the end of each month.
- Eric calculates that if he makes 48 deposits of R1 402,31, starting at the end of January 2024, he will have enough money to go on holiday. He will make his deposits into a savings account at the end of each month. The savings account pays interest at 8,6% p.a., compounded monthly.

Calculate the difference between the total amount that Eric and Thandi will deposit into their respective savings accounts over the given period. (4)

7.3 Lesibana was granted a loan of R225 000. The rate of interest for the loan is 9% p.a., compounded monthly. Lesibana will make monthly payments of R5 500, starting exactly four months after the loan was granted. How many payments will Lesibana make to settle the loan? (6)

[13]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x)=\frac{1}{x}$. (5)

8.2 Determine:

8.2.1 $\frac{d}{dx}\left(\sqrt{4x^6}+\sqrt{2}.x^2\right)$ (3)

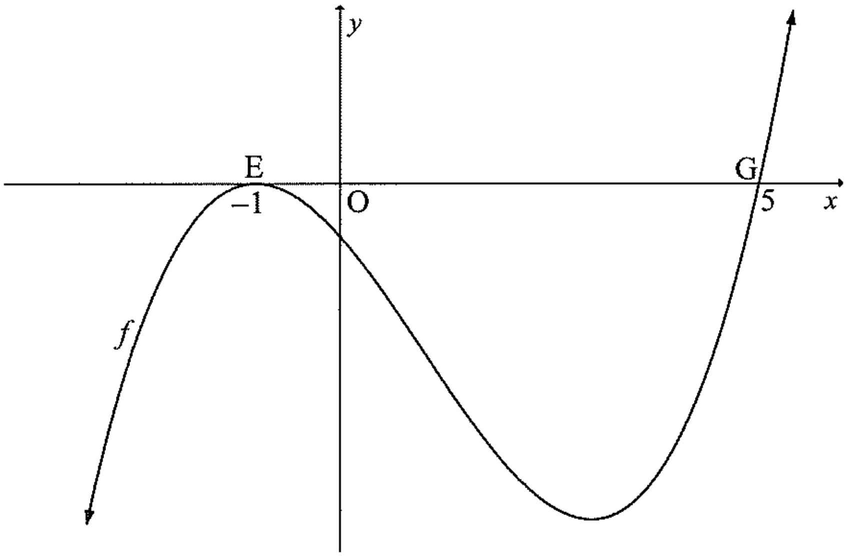
8.2.2 $g'(x)$ if $g(x)=\frac{3x^4-4x^2+6}{x^2}$ (3)

8.3 The equation of the tangent to $f(x)=3x^2+bx+c$ at $x=1$ is given by $y=9x-9$. Determine the values of b and c. (4)

[15]

QUESTION 9

The graph of $f(x) = ax^3 + bx^2 + cx - 5$ is drawn below. E(– 1 ; 0) and G(5 ; 0) are the x-intercepts of f.



9.1 Show that $a = 1$, $b = -3$ and $c = -9$. (3)

9.2 Calculate the value of x for which f has a local minimum value. (4)

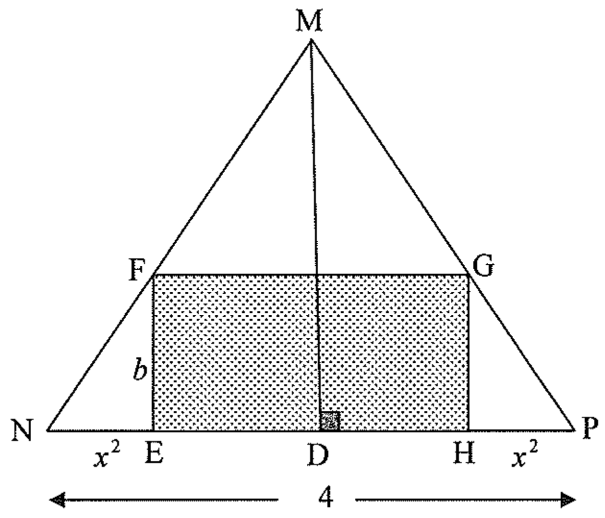
9.3 Use the graph to determine the values of x for which $f''(x).f(x) > 0$. (3)

9.4 For which values of t will the graph of $p(x) = f(x) + t$ have two distinct positive roots and one negative root? (3)

[13]

QUESTION 10

EHGF is a rectangle. HE is produced x^2 cm to N and EH is produced x^2 cm to P. NF produced intersects PG produced at M to form an isosceles triangle MNP with NM = MP. D lies on NP where MD $\perp$ NP. NP = 4 cm and MD = 3 cm.



10.1 Show that the area of EFGH is given by $A(x) = 6x^2 - 3x^4$. (4)

10.2 Calculate the maximum area of rectangle EFGH. (4)
[8]

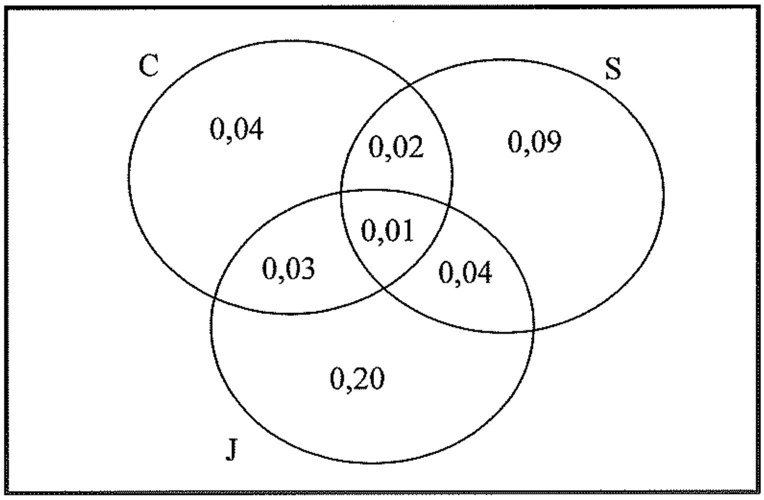
QUESTION 11

11.1 Two events, A and B, are such that:

- P(A) = 0,4
- P(A or B) = 0,52
- A and B are mutually exclusive

Calculate P(B). (2)

11.2 The items that a learner bought at a tuck shop were recorded over a period of time. The probabilities of the learner buying a sandwich (S), a chocolate (C) and a juice (J) are shown in the Venn diagram below.



11.2.1 What is the probability that the learner will buy a sandwich? (1)

11.2.2 Calculate the probability that the learner will buy at least two of the three items. (2)

11.2.3 Calculate the probability that the learner would NOT buy any of the three items. (2)

11.3 Seven guitar players, each with a different name, participate in a concert.

11.3.1 In how many different ways can the names of the guitar players be listed, one below the other, in the programme? (1)

11.3.2 After the performance, the guitar players wait backstage. There is a bench with only room for four to sit on.

What will be the probability that the four guitar players will be sitting in alphabetical order, from left to right? (3)

11.3.3 During the performance, the seven guitar players sit in a line on stage. Four guitar players are female and three are male.

In how many different ways can they be seated if the males may not sit next to each other? (3)

[14]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $A = P(1 - ni)$ $A = P(1 - i)^n$ $A = P(1 + i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r - 1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1 - (1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$$(x - a)^2 + (y - b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$